SET THEORY HOMEWORK 2

Due Monday, September 30.

Problem 1. *In ZF^- prove the Schröder-Bernstein theorem i.e. that if $A \preceq B$ and $B \preceq A$ implies that $A \approx B$.*
Hint: Suppose $f : A \to B$ and $g : B \to A$ are one-to-one.Set $A_0 = A$, $B_0 = B$, $A_{n+1} = g"B_n$, $B_{n+1} = f"A_n$, $A_\infty = \bigcap_n A_n$, $B_\infty = \bigcap_n B_n$. Let $h(x)$ be $f(x)$ if $x \in A_\infty \cup \bigcup_n (A_{2n} \setminus A_{2n+1})$. Otherwise let $h(x)$ be $g^{-1}(x)$. Show that h is well defined and $h : A \to B$ is one-to-one and onto.

Problem 2. *Assume CH (but not GCH). Show that for every natural number $n > 0$, $\omega_n^\omega = \omega_n$.*

Problem 3. (1) *Show that if $\aleph_\omega < 2^\omega$, then $(\aleph_\omega)^\omega = 2^\omega$.*
(2) *Show that $\aleph_\omega^{\omega_1} = 2^{\omega_1} \cdot \aleph_\omega^\omega$.*

Problem 4. *Suppose that for all n, $2^{\aleph_n} = \aleph_{\omega+1}$. Show that $2^{\aleph_\omega} = \aleph_{\omega+1}$.*
Hint: For each $A \subset \aleph_\omega$, define $A_n := A \cap \aleph_n$. Consider the map $A \mapsto \langle A_n \mid n < \omega \rangle$.

Problem 5. *Suppose that κ is inaccessible. Show that $|V_\kappa| = \kappa$ and $V_\kappa \cap ON = \kappa$.*